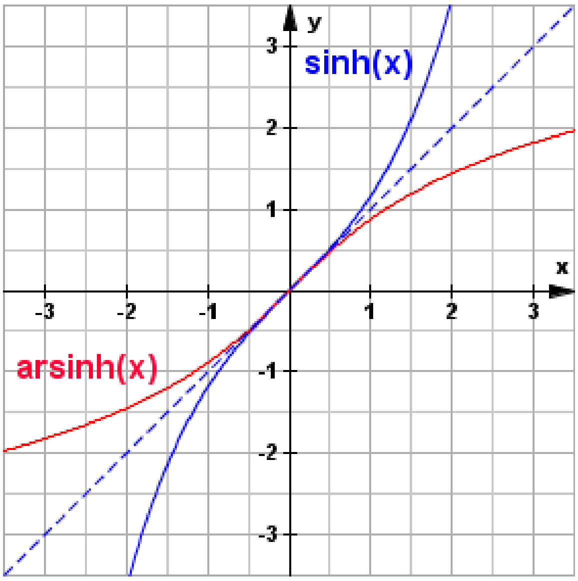
Hyperbel- und Areafunktionen

Ac 3-2017



Sinushyperbolicus:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Reihenententwicklung des Sinushyperbolicus:

$$\sinh(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$$

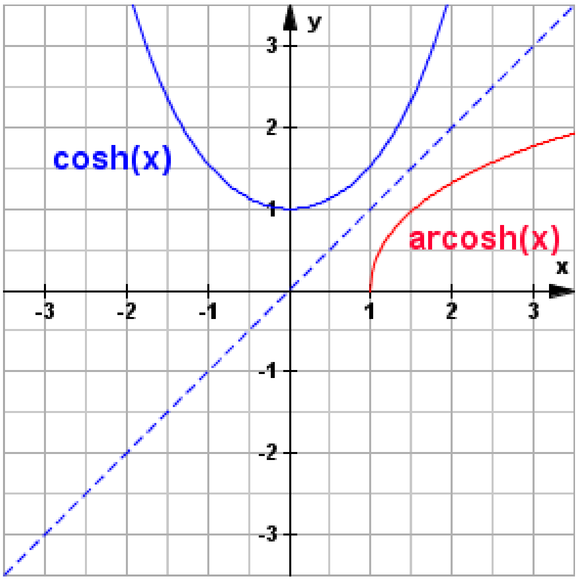
Ableitung: sinh(a·x)' = a·cosh(a·x)

Stammfunktion: $\int \sinh(a \cdot x)dx = \frac{1}{a}\cosh(a \cdot x) + C$

AreaSinushyperbolicus:

$$\operatorname{arsinh}(x) = \ln(x + \sqrt{x^2 + 1})$$

Ableitung: $\operatorname{arsinh}(x)' = \frac{1}{\sqrt{1 + x^2}}$



Cosinushyperbolicus:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Reihenententwicklung des Cosinushyperbolicus:

$$\cosh(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$$

Ableitung: cosh(a·x)' = a·sinh(a·x)

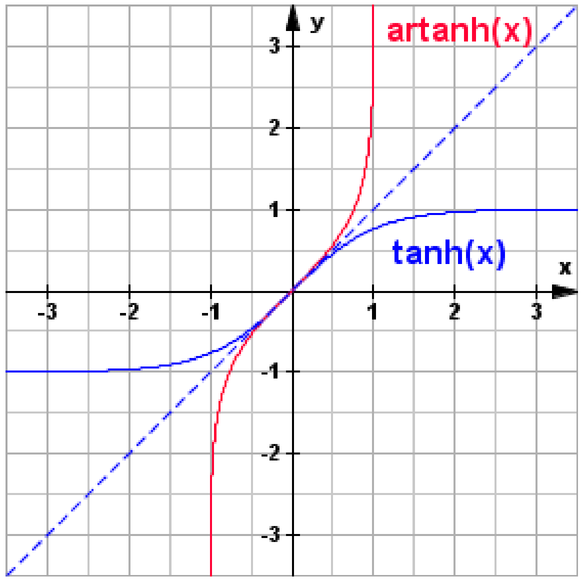
Stammfunktion: $\int \cosh(a \cdot x)dx = \frac{1}{a}\sinh(a \cdot x) + C$

cosh²(a·x) - sinh²(a·x) = 1

AreaCosinushyperbolicus:

$$\operatorname{arcosh}(x) = \ln(x + \sqrt{x^2 - 1}) \; ; \; |x| \geq 1$$

Ableitung: $\operatorname{arcosh}(x)' = \frac{1}{\sqrt{x^2 - 1}} \; ; \; x > 1$



Tangenshyperbolicus:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Reihenententwicklung des Tangenshyperbolicus:

$$\tanh(x) = \sum_{k=0}^{n} \frac{(-1)^{k+1} 2^k (2^k - 1)}{(2k)!} B_k x^{2k-1}$$

Ableitung: tanh(a·x)' = a / cosh²(a·x)

Stammfunktion: $\int \tanh(a \cdot x) dx = \frac{1}{a} \ln(\cosh(a \cdot x)) + C$

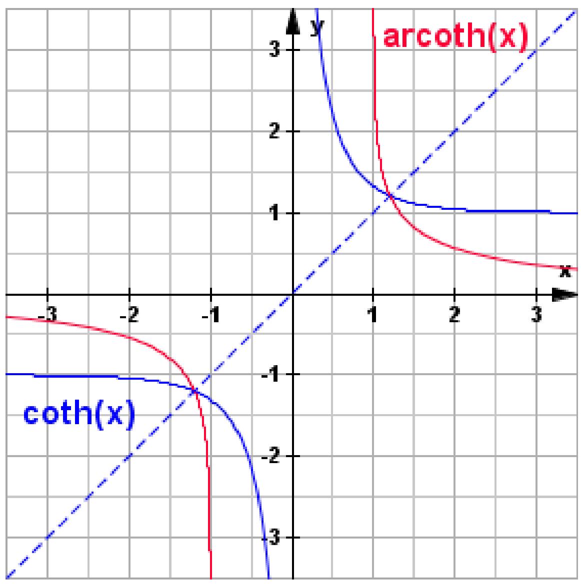
AreaTangenshyperbolicus:

$$\operatorname{artanh}(x) = \frac{1}{2} \ln(\frac{1+x}{1-x}) \ ; \ |x| < 1$$

Reihenentwicklung des AreaTanhyperbolicus:

$$\operatorname{artanh}(x) = \sum_{k=0}^{n} \frac{x^{2k+1}}{2k+1} = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots$$

Ableitung: $\operatorname{artanh}(x)' = \frac{1}{1-x^2} \ ; \ |x| < 1$



Cotangenshyperbolicus:

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{1}{\tanh(x)}$$

Reihenentwicklung des Cotangenshyperbolicus:

$$\coth(x) = \sum_{k=0}^{n} \frac{(-1)^{k+1} 2^k}{(2k)!} B_k x^{2k-1}$$

Ableitung: coth(a·x)' = - a / sinh²(a·x)

Stammfunktion: $\int \coth(a \cdot x) dx = \frac{1}{a} \ln(\sinh(a \cdot x)) + C$

AreaCotangenshyperbolicus:

$$\operatorname{arcoth}(x) = \frac{1}{2} \ln(\frac{1+x}{x-1}) \ ; \ |x| > 1$$

Ableitung: $\operatorname{arcoth}(x)' = \frac{1}{1-x^2} \ ; \ |x| > 1$